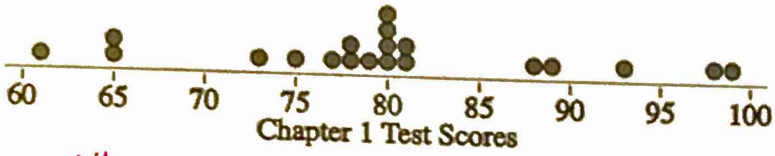
Lesson 2.1: How Did I Do?

How well did you do on the Chapter 1 Test? How well did you do relative to your classmates?

Here are the results of a random sample of 20 of the Chapter 1 Tests, along with a dotplot and summary statistics.

Test Scores	61	65	65	73	75	77	78	78	79	80	80	80	80	81	81	88	89	93	98	99



n	mean	SD	min	Q1	med	Q3	max
20	80	10	61	76	80	84.5	99

Percentile: percent of values strictly less or equal to.

1. Biff scored a 65. What is Biff's percentile? $\frac{3}{20} = 0.15$ 15th percentile

2. Was Biff above or below the mean? By how many points? By how many standard deviations?

$Z = -1.5$ Below. 15 points below mean. 1.5 standard deviations below the mean.

Below mean → negative z-score.

3. Marty scored an 88. What is Marty's percentile? $\frac{16}{20} = 0.80$ 80th percentile

4. Was Marty above or below the mean? By how many points? By how many standard deviations?

$Z = 0.8$ Above. 8 points above mean. 0.8 standard deviations above the mean.

Above mean → positive z-score.

A z-score is defined as the number of standard deviations above or below the mean.

5. Write a formula for calculating a z-score. $z = \frac{\text{Value} - \text{mean}}{SD}$

6. Goldie scored a 98 on the Chapter 1 Test. Find and interpret the z-score.

$Z = \frac{98-80}{10} = \frac{18}{10} = 1.8$

Goldie's Chapter 1 test score is 1.8 standard deviations above the mean.

Bonus: Goldie was aspiring for what job?

Mayor (Back to the Future)

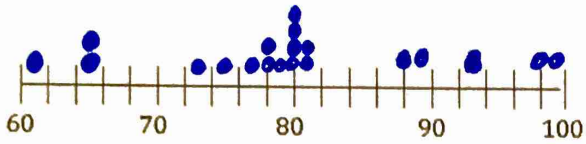
↑ Who takes this job?

7. There are two mathematical operations used when calculating a z-score:

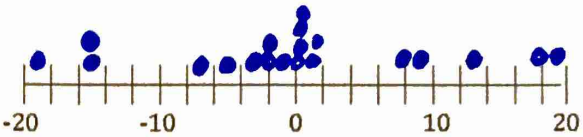
a. First, we take each score, and Subtract the mean (remember the mean was 80). Fill in the table and then make a dotplot for each.

SCORE	61	65	65	73	75	77	78	78	79	80	80	80	80	81	81	88	89	93	98	99
SCORE - MEAN	-19	-15	-15	-7	-5	-3	-2	-2	-1	0	0	0	0	1	1	8	9	13	18	19

Dotplot for SCORE



Dotplot for SCORE - MEAN



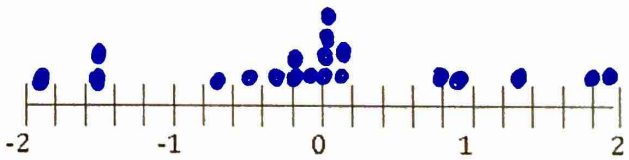
What happens to the shape, center, and variability when you subtract the mean from each score?

Same — shifted down 80 ↳ New mean = 0 — Same — Check range: 99-61 = 38, 19--19 = 38

b. Second, we take the SCORE – MEAN and divide by the standard deviation (remember the standard deviation is 10). Fill in the table and then make a dotplot for each.

SCORE - MEAN	-19	-15	-15	-7	-5	-3	-2	-2	-1	0	0	0	0	1	1	8	9	13	18	19
SCORE – MEAN / SD	-1.9	-1.5	-1.5	-.7	-.5	-.3	-.2	-.2	-.1	0	0	0	0	.1	.1	.8	.9	1.3	1.8	1.9

Dotplot for $\frac{SCORE - MEAN}{SD}$



What happens to the shape, center, and variability when you divide by the standard deviation for each value?

Same — same at 0 (↑ yes but actually was ÷10 (0/10 = 0)) — decreased (÷ by 10) → New SD: 10/10 = 1

8. Summarize: What happens to the shape, center and variability of a distribution when you

...add or subtract the same value *a* from each value?

Shape → same Center → +/- a Variability → Same

...multiply or divide by the same value *b* from each value?

Shape → same Center → ×/÷ b Variability → ×/÷ b

9. What is the mean and standard deviation of the distribution of z-scores? Will this be true for any distribution of z-scores? Explain.

Mean = 0 SD = 1

Standardized distribution (z-scores) always have mean = 0 and SD = 1.

Lesson 2.1 – Describing Location in a Distribution

Big Ideas:

LT#1: Standardized values (z-scores)

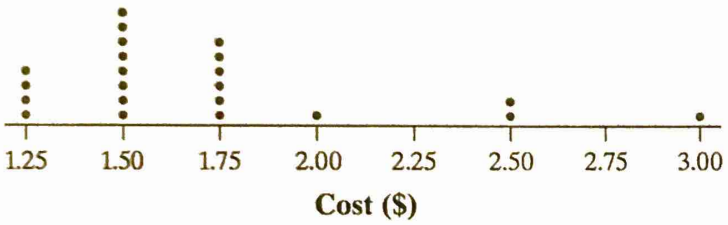
$z = \frac{\text{Value} - \text{mean}}{SD}$

Context is z standard deviations above or below the mean.

LT#2: Given any list of #'s
- Add/subtract a number a to each value...
  Shape & variability stay the same
  Center shifts up (down) by a.
- multiply/divide by a number b...
  Shape stays the same, center & variability mult. (divide) by b.

Check Your Understanding:

Knoebels Amusement Park in Elysburg, Pennsylvania, has earned acclaim for being an affordable, family-friendly entertainment venue. Knoebels does not charge for general admission or parking, but it does charge customers for each ride they take. How much do the rides cost at Knoebels? The figure shows a dot- plot of the cost for each of 22 rides in a recent year, along with summary statistics.



n	Mean	SD	Min	Q_1	Median	Q_3	Max
22	1.705	0.447	1.25	1.5	1.5	1.75	3

1. Suppose you convert the cost of the rides from dollars to cents ($1=100 cents). Describe the shape, mean, and standard deviation of the distribution of ride cost in cents.

   Same (skewed right)

   multiplied by 100
   $1.705 \times 100 = 170.5$ cents

   multiplied by 100
   $0.447 \times 100 = 44.7$ cents

2. Knoebels' managers decide to increase the cost of each ride by 25 cents. How would this the shape, center, and variability of this distribution compare with the distribution of cost in Question 1?

   Same

   Shifts up 25
   $170.5 + 25 = 195.5$ cents

   Same

3. Now suppose you convert the increased costs from Question 2 to z-scores. What would be the shape, mean, and standard deviation of this distribution? Explain your answers.

   Same

   0

   1

   All standardized distributions have a mean = 0 and SD = 1.